To understand the distortion of $\mathbf{Q}_e$ due to anisotropy, we examine the anisotropic contribution to the canonical battery term, which is equal to

$$\begin{aligned} -\nabla \times \left(\nabla \cdot \mathbf{p}_{e,aniso}/n_e\right) &\simeq -\nabla \times \left(\nabla \cdot \left[\sigma \mathbf{B}\mathbf{B}\right]\right) \\ &= -\nabla \times \left(\mathbf{B}\left[\mathbf{B} \cdot \nabla \sigma\right] + \sigma \mathbf{B} \cdot \nabla \mathbf{B}\right). \end{aligned} \tag{S1}$$

Since an increase of Q_{ey} shear corresponds to an increase in u_{ez}, we consider the y-component of the canonical induction equation (Eq. 4 of the paper) and thus of Eq. S1 which is $\partial \left(\mathbf{B}\left[\mathbf{B} \cdot \nabla \sigma\right] + \sigma \mathbf{B} \cdot \nabla \mathbf{B}\right)_z / \partial x = \partial \left(B_z \left[\mathbf{B} \cdot \nabla \sigma\right] + \sigma \mathbf{B} \cdot \nabla B_z\right) / \partial x$. Here, σ depends on $|\mathbf{B}|$ and B_z is of the same order of magnitude as B_y. Therefore, since B_y has the shortest spatial scale of the components of $\mathbf{B}$, it follows that $\left|\nabla \sigma\right| / \sigma \gg \left|\nabla B_z / \right| B_z$, so $\partial \left(B_z \left[\mathbf{B} \cdot \nabla \sigma\right] + \sigma \mathbf{B} \cdot \nabla B_z\right) / \partial x \simeq \partial \left(B_z \left[\mathbf{B} \cdot \nabla \sigma\right]\right) / \partial x \simeq B_z \partial \left[\mathbf{B} \cdot \nabla \sigma\right] / \partial x$.

Figure S1 shows various quantitites involved in the calculation of $-\hat{y} \cdot \nabla \times \left(\nabla \cdot \mathbf{p}_{e,aniso}/n_e\right) \simeq B_z \partial \left[\mathbf{B} \cdot \nabla \sigma\right] / \partial x$. The quadrupole out-of-plane Hall fields (plus and minus signs in Fig. 1 of the paper) add and subtract to the background guide field B_z prescribed by Eq. 11 in the paper. This, together with the reduction of $|\mathbf{B}|$ due to reconnection, generates a region of low $|\mathbf{B}|$ (Fig. S1a) with a pronounced tilt. Because $p_{\|} \sim n_e^3/B^2$ and $p_{\perp} \sim n_e B$, this $|\mathbf{B}|$ tilt generates a tilted region of finite σ (Fig. S1b; color). The quantity $\mathbf{B} \cdot \nabla \sigma$, which is the variation of σ along the in-plane $\mathbf{B}$ (Fig. S1b; lines), is shown in Fig. S1c (color), and its gradient $\nabla \left(\mathbf{B} \cdot \nabla \sigma\right) \simeq \hat{x} \partial \left[\mathbf{B} \cdot \nabla \sigma\right] / \partial x$ is represented by the black arrows. The resultant $-\hat{y} \cdot \nabla \times \left(\nabla \cdot \mathbf{p}_{e,aniso}/n_e\right) \simeq B_z \partial \left[\mathbf{B} \cdot \nabla \sigma\right] / \partial x$ is shown in Fig. S1d (contour). The convective term $\hat{y} \cdot \nabla \times \left(\mathbf{u}_e \times \mathbf{Q}_e\right)$ (Fig. S1d; color) enhances Q_{ey} shear (Fig. S1d; red arrows) as expected from Eq. 5 in the paper, and $-\hat{y} \cdot \nabla \times \left(\nabla \cdot \mathbf{p}_{e,aniso}/n_e\right)$ further enhances this shear in spatial extent in the y direction. The resultant $\partial Q_{ey}/\partial t = \hat{y} \cdot \nabla \times \left(\mathbf{u}_e \times \mathbf{Q}_e\right) - \hat{y} \cdot \nabla \times \left(\nabla \cdot \mathbf{p}_{e,aniso}/n_e\right)$ is shown in Fig. S1e; this increase in Q_{ey} shear is the origin of the elongated out-of-plane flow u_{ez}.

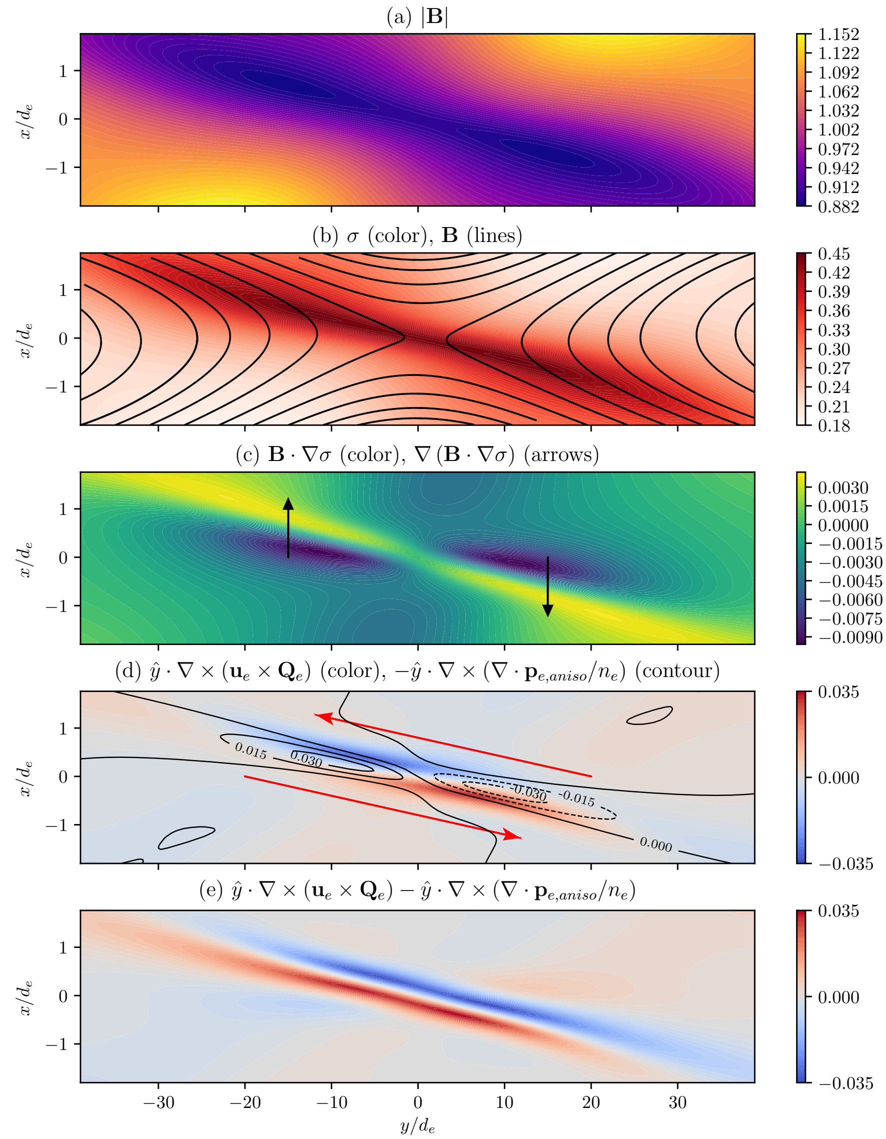

Figure S1: (a-c) Various quantities involved in the calculation of $-\hat{y}\cdot\nabla\times(\nabla\cdot\mathbf{p}_{e,aniso}/n_e)\simeq B_z\partial\left[\mathbf{B}\cdot\nabla\sigma\right]/\partial x$ for the simulation corresponding to Fig. 3b in the paper. (d) The y-component of the convective term $\hat{y}\cdot\nabla\times(\mathbf{u}_e\times\mathbf{Q}_e)$ and the anisotropic contribution to the canonical battery term $-\hat{y}\cdot\nabla\times(\nabla\cdot\mathbf{p}_{e,aniso}/n_e)$, and (e) their sum.